Maths Foundation
Seminar 3: Introduction to integration - Solution
Stockholm School of Economics in Riga, 2022

Exercise 1

1. $\int x^7 dx = \frac{1}{8}x^8 + C$

2. $\int x^{3/2} dx = \frac{2}{5}x^{5/2} + C$

3. $\int 6x^5 dx = x^6 + C$

4. $\int 8x^3 - 4x^2 + 5dx = 2x^4 - \frac{4}{3}x^3 + 5x + C$

5. $\int \sqrt[3]{x^4} dx = \int (x^4)^{1/3} dx = \int x^{4/3} dx = \frac{3}{7}x^{7/3} + C$

6. $\int x\sqrt{x} dx = \int xx^{1/2} = \int x^{3/2} = \frac{2}{5}x^{2/5} + C$

7. $\int \sqrt{\sqrt{\sqrt{x}}} dx = \int ((x^{1/2})^{1/2})^{1/2} dx = \int x^{1/8} dx = \frac{8}{9}x^{9/8} + C$

Exercise 2

The revenue function is:
$\int 12\sqrt[3]{x} + 3\sqrt{x} dx = 3 \int 4x^{1/3} + x^{1/2} dx = 3(3x^{4/3} + \frac{2}{3}x^{3/2}) + C$
$= 9x^{4/3} + 3x^{3/2} + C$.
Logically, if the company sells nothing, its revenue is null. Accordingly, $R(0) = 9x^{4/3} + 3x^{3/2} + C = 0$, so $C = 0$.

Exercise 3

The price p of the flat in the future is given by: $p(t) = \int 600\sqrt{t} dt = 400t^{3/2} + C$.
At $t = 0$, the flat worth 200000 euros, so $p(0) = 400t^{3/2} + C = 200000$. Hence, the price in 25 years will be $p(25) = 200000 + 400 \times 25^{3/2} = 250000$ euros.

Exercise 4

1. $\int_0^2 x^3 dx = \Big|_0^2 \frac{1}{4}x^4 = \frac{1}{4} \times 2^4 - \frac{1}{4} \times 0^4 = 4$
2. $\int_{-1}^1 1 - x^2 dx = \Big|_{-1}^1 x - \frac{1}{3}x^3 = 1 - \frac{1}{3} - (-1 + \frac{1}{3}) = \frac{4}{3}$
3. $\int_4^9 \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = \Big|_4^9 2x^{1/2} = 2 \times 9^{1/2} - 2 \times 4^{1/2} = 2$
4. $\int_1^2 6x^2 + 4x - 1 dx = \Big|_1^2 2x^3 + 2x^2 - x = 16 + 8 - 2 - 2 - 2 + 1 = 19$

Exercise 5

- $\int x^2 dx = \frac{1}{3}x^3 + C.$
- $\int_0^3 x^2 dx = \Big|_0^3 (\frac{1}{3}x^3 + C) = \frac{1}{3} \times 3^3 + C - \frac{1}{3} \times 0 - C = 9.$
- The constant just cancels out!

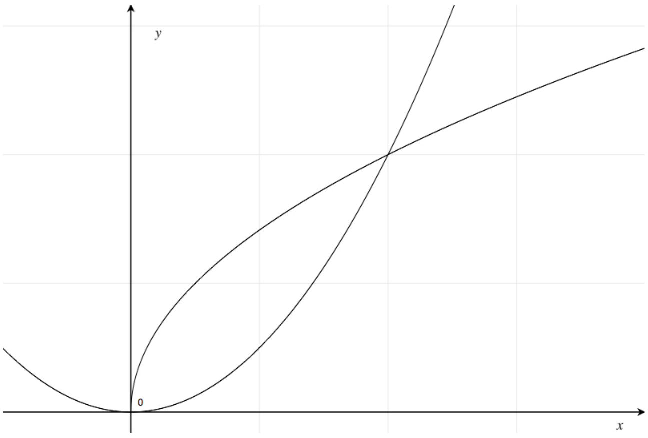
Exercise 6

1. The area can be computed as: $\int_a^b h dx = \Big|_a^b hx = bh - ah = h(b - a).$
2. First, note that the slope of the line is $\frac{h-0}{a-0} = \frac{h}{a}$.
   We can thus write: $\int_0^a \frac{a}{h} x dx = \frac{h}{a} \Big|_0^a \frac{1}{2}x^2 = \frac{h}{a} (\frac{1}{2}a^2 = \frac{ah}{2}.$

Exercise 7

$\int_1^5 30t^{-2} dt = 30 \Big|_1^5 - t^{-1} = 30 \times [-\frac{1}{5} + 1] = 24.$
Between the beginning of January and the beginning of May, there will be 24 guitars sold.

Exercise 8



We are looking for the area which is under y_1 and above y_2. We can note that this area corresponds to the 'full' area under y_1 minus the area under y_2.
Also, we need to determine what are the values of x for which we have $\sqrt{x} = x^2$. This equality holds for $x = 0$ and $x = 1$.
We thus have:
$\int_0^1 \sqrt{x} - x^2 dx = \Big|_0^1 \frac{2}{3}x^{3/2} - \frac{1}{3}x = \frac{1}{3}.$