

# SSE Riga - Maths Foundation

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February 5, 2022



- 3 sessions :
  - February 5
  - February 12
  - February 19
- Starts at 10 :00
- Lecture + seminar
- Lecture slides + problem sets + solutions available online
- Certificate of attendance for students attending **all** three lectures

- Session 1 : Introduction to differentiation
- Session 2 : Introduction to optimization
- Session 3 : Introduction to integral calculus

# Introduction to differentiation

## Definition

A function  $f$  is a rule that assigns to each number  $x$  in a set a number  $f(x)$ . The set of all allowable values of  $x$  is called the domain, and the set of all values  $f(x)$  for  $x$  in the domain is called the range

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In economics :

- $Q_D(P)$  : demand function
- $U(x)$  : utility function
- $\Pi(Q)$  : profit function
- ...

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What is the **rate of change**?



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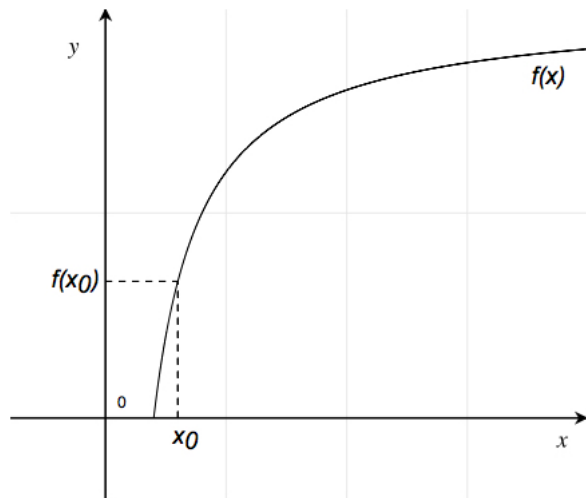
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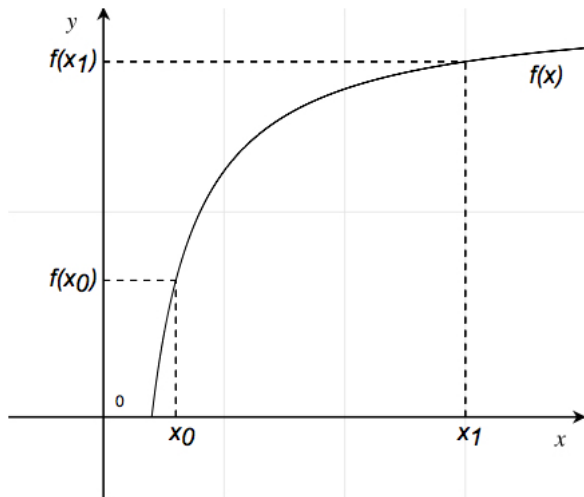
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- Example :  $Q_d(P) = -0,15P + 0,14$  represents the demand function for chocolate, with  $P$  in euro and  $Q$  in kg.

How to measure the rate of change when the function is not linear ?

What is the slope at  $x = x_0$ ?

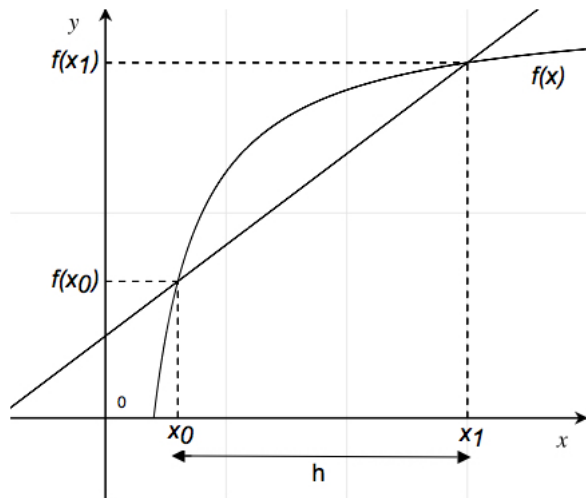


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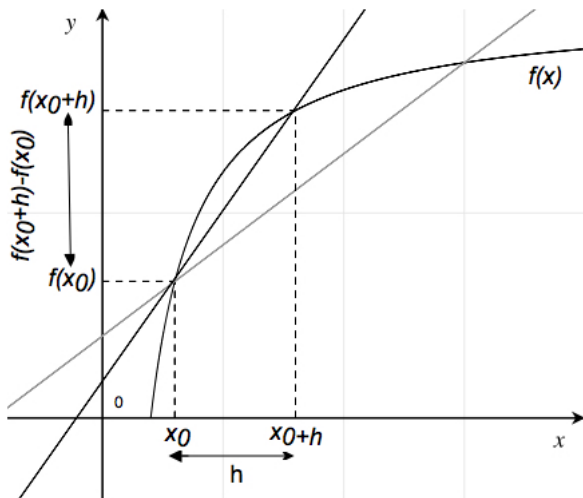




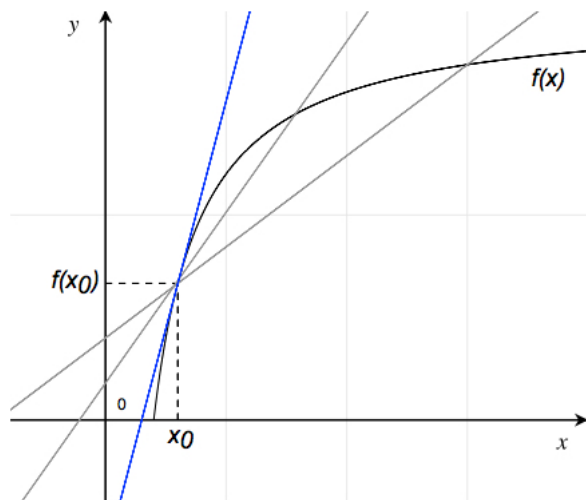
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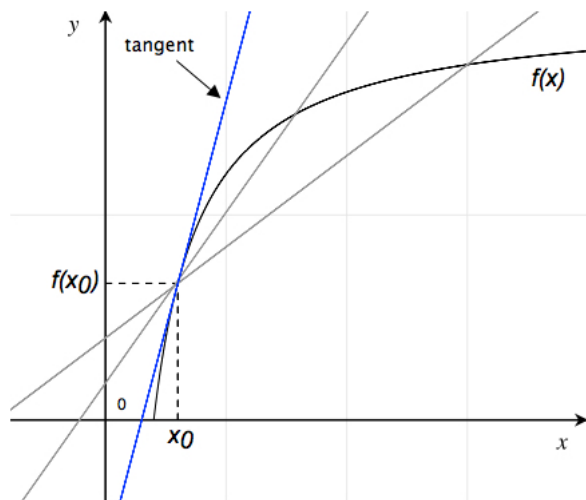
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# This is the most important slide of your life

## definition

Let  $(x_0, f(x_0))$  be a point on the graph of  $y = f(x)$ . The **derivative** of  $f$  at  $x_0$  is the slope of the tangent line to the graph of  $f$  at  $(x_0, f(x_0))$ . We write :

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

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- Power rule :

$$\text{If } f(x) = x^a, \text{ then } f'(x) = ax^{a-1}$$



# Differentiation of sums and differences

Consider the two differentiable functions  $u(x)$  and  $v(x)$

- If  $f = u + v$ , then  $f' = u' + v'$
- If  $f = u - v$ , then  $f' = u' - v'$

- Next week : introduction to optimization



Thank you for your attention !

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